

Proximity-induced screening and its magnetic breakdown in mesoscopic hybrid structures

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We derive a general microscopic expression for the non-linear diamagnetic current in a clean superconductor-insulator-normal metal structure with an arbitrary interface transmission. In the absence of electron-electron interactions in the normal metal the diamagnetic response increases monotonously with decreasing temperature showing no sign of paramagnetic reentrance down to $T = 0$. We also analyze the magnetic breakdown of proximity-induced Meissner screening. We demonstrate that the magnetic breakdown field should be strongly suppressed in the limit of small interface transmissions while the linear diamagnetic current does not depend on the transmission of the insulating barrier at low enough temperatures.

I. INTRODUCTION

Mesoscopic hybrid structures composed of superconducting (S) and normal (N) metals demonstrate a rich variety of intriguing physical phenomena. Many of them have recently been extensively investigated – both experimentally and theoretically – and received adequate interpretation within the framework of the quasiclassical theory of superconductivity, see, e.g., Refs. 1,2 for a recent review.

One of the remaining puzzles in the field concerns the paramagnetic reentrance phenomenon observed in silver-niobium^{3,4} and gold-niobium⁵ proximity cylinders in the limit of very low temperatures. This reentrance behavior is in a clear contradiction with earlier theoretical predictions⁶ as well as with the results of more recent studies⁷ demonstrating that diamagnetism in SN proximity cylinders should progress *monotonously* with decreasing temperature. Several theoretical explanations of this reentrance phenomenon have been put forward^{8,9,10}. However, a detailed comparison with experiments carried out in Refs. 4,5 indicates that none of these explanations is able to correctly reproduce the absolute value or the temperature dependence of the observed paramagnetic reentrance effect. One of the explanations has also been subject to theoretical debate¹¹.

It is important to emphasize that, while the authors^{9,10} invoke additional assumptions about the form of electron-electron interactions in the N-metal, the work⁸ operates with the standard model for SN proximity systems which assumes no interaction between electrons in the normal layer. This model is usually very well described by means of the standard approach based on the quasiclassical Eilenberger equations. In order to test the conjecture⁸, that paramagnetic reentrance could be missing within the standard quasiclassical formalism because of an additional contribution from the low energy “glancing” states, one needs to go beyond quasiclassics and analyze the problem within a more general approach. Below we will use the microscopic Gor’kov equations and eval-

uate the screening current by constructing the full Green functions for the problem in question. We will then compare our result with that derived from the Eilenberger equations. This comparison confirms the applicability of the standard quasiclassical technique and demonstrates that no paramagnetic reentrance can occur in SN proximity cylinders in the absence of electron-electron interactions in the N-layer.

Another interesting property of the same structures is the so-called magnetic breakdown of proximity-induced Meissner screening. This effect occurs at a certain value of the external magnetic field H_b , above which the diamagnetic expulsion of the field from the normal layer turns out to be energetically unfavorable^{7,12,13} although the proximity effect itself is not yet suppressed. In the absence of impurities in the N-layer and for the case of a perfectly transparent interface between S- and N-metals the breakdown field $H_b(T)$ was evaluated in Ref. 13. Since at sufficiently low temperatures the *linear* current response of SN proximity cylinders does not depend on the transmission of the SN interface¹⁴ one could assume that at $T \rightarrow 0$ the result¹³ for H_b should also apply at arbitrary transmissions. However, this turns out not to be the case. As we will show below, the breakdown field scales linearly with the interface transmission and, hence, H_b should be strongly suppressed in the limit of small transmissions. This effect makes it possible to extract information about the interface transmission from the measurements of the magnetic breakdown field H_b .

Our paper is organized as follows. In section II we will present a general derivation of the non-linear screening current in superconductor-normal metal proximity systems with arbitrary interface transmissions. This derivation is carried out on the basis of the microscopic Gor’kov equations and does not involve energy-integrated quasiclassical Eilenberger functions. The effect of magnetic breakdown of proximity-induced screening is analyzed in section III. Applicability of the quasiclassical approach for the problem in question is discussed in section IV.

II. NONLINEAR DIAMAGNETIC RESPONSE

We will consider a clean SIN structure and assume it to be uniform along the directions parallel to the interfaces (coordinates y and z). The normal metallic layer (N) and the bulk superconductor (S) are located respectively at $0 < x \leq d$ and $x < 0$. Our analysis will be based on the Gor'kov equations of the microscopic theory of superconductivity¹⁶. After the Fourier transformation of the normal (G) and anomalous (F^+) Green functions with respect to y and z ,

$$G_{\omega_n}(\mathbf{r}, \mathbf{r}') = \int \frac{d^2 \mathbf{k}_{\parallel}}{(2\pi)^2} G_{\omega_n}(x, x', \mathbf{k}_{\parallel}) e^{i\mathbf{k}_{\parallel}(\mathbf{r}_{\parallel} - \mathbf{r}'_{\parallel})},$$

the Gor'kov equations take the following standard form

$$\begin{pmatrix} i\omega_n - \hat{H} & \Delta(x) \\ \Delta^*(x) & i\omega_n + \hat{H}_c \end{pmatrix} \begin{pmatrix} G_{\omega_n}(x, x', \mathbf{k}_{\parallel}) \\ F_{\omega_n}^+(x, x', \mathbf{k}_{\parallel}) \end{pmatrix} = \begin{pmatrix} \delta(x - x') \\ 0 \end{pmatrix} \quad (1)$$

Here $\omega_n = (2n + 1)\pi T$ is the Matsubara frequency, and $\Delta(x)$ is the superconducting order parameter. Below we will choose $\Delta(x) = \Delta$ for $x < 0$ and $\Delta(x) = 0$ otherwise. The Hamiltonian \hat{H} in Eq. (1) is defined as

$$\hat{H} = -\frac{1}{2m} \frac{\partial^2}{\partial x^2} + \frac{\tilde{\mathbf{k}}_{\parallel}^2}{2m} - \epsilon_F + V(x), \quad (2)$$

where $\tilde{\mathbf{k}}_{\parallel} = \mathbf{k}_{\parallel} - \frac{e}{c} \mathbf{A}_{\parallel}(x)$, ϵ_F is Fermi energy and the term $V(x)$ accounts for the boundary potentials. The Hamiltonian \hat{H}_c is obtained from \hat{H} (2) by inverting the sign of the electron charge e .

In what follows we shall neglect the square term $\propto \mathbf{A}_{\parallel}^2$ in Eq. (2). This approximation applies in the range of magnetic fields $H \ll \Phi_0 k_F/d$, where k_F is the Fermi momentum and $\Phi_0 = \pi c/e$ is the superconducting flux quantum. We will assume that the London penetration depth of the bulk superconductor is small and neglect the magnetic field inside the superconductor. The latter assumption allows to set $\mathbf{A}(x \leq 0) = 0$.

The next standard step is to decompose the Green functions into the product of quickly oscillating terms $\exp(\pm i k_x x)$ and the two component envelope functions $\bar{\varphi}_{\pm}(x)$ changing at scales much longer than the Fermi wavelength. One has

$$\begin{pmatrix} i\omega_n - \hat{H} & \Delta(x) \\ \Delta^*(x) & i\omega_n + \hat{H}_c \end{pmatrix} \bar{\varphi}_{\pm}(x) e^{\pm i k_x x} \simeq e^{\pm i k_x x} \begin{pmatrix} i\omega_n - \hat{H}_{\pm}^a & \Delta(x) \\ \Delta^*(x) & i\omega_n + \hat{H}_{\pm c}^a \end{pmatrix} \bar{\varphi}_{\pm}(x), \quad (3)$$

where we defined $k_x = \sqrt{k_F^2 - k_{\parallel}^2}$ and

$$\hat{H}_{\pm}^a = \mp i v_x \partial_x - \frac{e}{c} \mathbf{A}_{\parallel}(x) v_{\parallel}. \quad (4)$$

A convenient choice of the gauge for the problem in question is $\mathbf{A} = (0, A(x), 0)$. Below we will make use of

this gauge and proceed along the lines with our previous analysis¹⁵.

Let us fix the coordinate x' within the normal metal. In this case for $x < 0$ the general solution of Eq. (1) can be written in the form

$$\begin{pmatrix} G_{\omega_n}(x, x') \\ F_{\omega_n}^+(x, x') \end{pmatrix} = \begin{pmatrix} 1 \\ -i \end{pmatrix} e^{\Delta x/v_x} e^{i k_x x} s_1(x') \quad (5) \\ + \begin{pmatrix} 1 \\ i \end{pmatrix} e^{\Delta x/v_x} e^{-i k_x x} s_2(x').$$

Here we restricted ourselves to small Matsubara frequencies $\omega_n \ll \Delta$. This is sufficient in the most interesting physical limit $T \ll \Delta$ and $d \gg \xi_0 \sim v_F/\Delta$ which we only consider below.

The general solution of Eq. (1) for $x > 0$ reads

$$\begin{pmatrix} G_{\omega_n}(x, x') \\ F_{\omega_n}^+(x, x') \end{pmatrix} = \begin{pmatrix} \tilde{G}_{\omega_n}(x, x') \\ 0 \end{pmatrix} + \quad (6) \\ + \begin{pmatrix} \varphi_+(x) e^{i k_x x} f_1(x') + \varphi_-(x) e^{-i k_x x} f_2(x') \\ \varphi_-(x) e^{i k_x x} f_3(x') + \varphi_+(x) e^{-i k_x x} f_4(x') \end{pmatrix}.$$

Here the functions

$$\varphi_+(x) = \exp\left(-\frac{\omega_n}{v_x} x + i \int_0^x \frac{e k_y}{c k_x} A(x_1) dx_1\right), \quad (7) \\ \varphi_-(x) = \frac{1}{\varphi_+(x)}.$$

obey the equations $\hat{H}_{\pm}^a \varphi_{\pm}(x) = 0$ and the first term in the right hand side of (6) with

$$\tilde{G}_{\omega_n}(x, x') = -\frac{i}{v_x} \begin{cases} \varphi_+(x) \varphi_-(x') e^{i k_x (x - x')} & \text{if } x > x' \\ \varphi_-(x) \varphi_+(x') e^{i k_x (x' - x)} & \text{if } x < x' \end{cases} \quad (8)$$

represents the particular solution of Eq. (1) for $x > 0$. What remains is to determine six functions $s_{1,2}(x')$ and $f_{1-4}(x')$ in Eqs. (5,6). This is done with the aid of the boundary conditions imposed on both sides of the normal layer.

First we consider the SN interface. Matching the wave functions on both sides of this interface, respectively $A_1 \exp(i k_{1x} x) + B_1 \exp(-i k_{1x} x)$ and $A_2 \exp(i k_{2x} x) + B_2 \exp(-i k_{2x} x)$, is performed in a standard manner (see, e.g., Ref. 17):

$$A_2 = \alpha A_1 + \beta B_1, \quad B_2 = \beta^* A_1 + \alpha^* B_1, \quad (9) \\ |\alpha|^2 - |\beta|^2 = 1,$$

where the reflection and transmission coefficients are equal to

$$R = \left| \frac{\beta}{\alpha} \right|^2, \quad D = 1 - R = \frac{1}{|\alpha|^2}. \quad (10)$$

Applying Eqs. (9) to the two-component vectors (G, F^+) we get four linear equations for six unknown functions in Eqs. (5,6). The remaining two equations are derived

at the interface between vacuum and the normal metal ($x = d$) where we assume complete specular reflection of the electrons. This boundary condition trivially yields

$$G(d, x') = F^+(d, x') = 0. \quad (11)$$

The above six linear equations uniquely define the Green function of our system. Technically the calculation is similar to that presented in Ref. 15, therefore we will not go into further details here. The resulting expression for the Green function in the N-metal ($0 < x \leq d$) takes the form

$$G_{\omega_n}(x, x, \mathbf{k}_{\parallel}) = -\frac{1}{v_x} \left[\frac{i \sinh \chi + \frac{2\sqrt{R}}{1+R} \sin(\gamma)}{\cosh \chi + \frac{2\sqrt{R}}{1+R} \cos(\gamma)} \right], \quad (12)$$

$$\chi = \frac{2\omega_n d}{v_x} - 2i \int_0^d \frac{ek_y}{ck_x} A(x) dx, \quad \gamma = 2k_x d + \arg \frac{\beta}{\alpha^*}.$$

Of interest for us here is the current density in the y -direction. It reads

$$j(x) = \frac{4e}{m} T \sum_{\omega_n > 0} \int_{|\mathbf{k}_{\parallel}| < k_F} \frac{d^2 \mathbf{k}_{\parallel}}{(2\pi)^2} k_y \text{Re} G_{\omega_n}(x, x, \mathbf{k}_{\parallel}). \quad (13)$$

Note, that the terms $\sin \gamma$ and $\cos \gamma$ in Eq. (12) are quickly oscillating functions of \mathbf{k}_{\parallel} . Integration over the momentum directions in (13) is equivalent¹⁵ to averaging over the angle γ . Performing this averaging we arrive at a spatially homogeneous screening current in the N-layer

$$j = \frac{4ek_F^2 T}{\pi^2} \sum_{\omega_n > 0} \int_0^{\pi/2} d\theta \int_0^{\pi/2} d\varphi \sin^2 \theta \cos \varphi \text{Im} G(\chi). \quad (14)$$

The function G in Eq. (14) depends on the complex variable $\chi = \chi_1 - i\chi_2$, where

$$\chi_1 = \frac{2\omega_n d}{v_F \cos \theta}, \quad \chi_2 = \phi \tan \theta \cos \varphi \quad (15)$$

and

$$\phi = \frac{2\pi}{\Phi_0} \int_0^d A(x) dx. \quad (16)$$

The function $G(\chi)$ is π -periodic in χ_2 . Within one period it is defined on a strip $-\pi/2 < \chi_2 < \pi/2$ with the cut going from $(0, -\arcsin t)$ to $(0, \arcsin t)$. For such values of χ_2 one has

$$G = \frac{\sinh \chi}{\sqrt{t^2(\theta) + \sinh^2 \chi}}, \quad (17)$$

where $t(\theta) = D(\theta)/(1 + R(\theta))$. Eqs. (14)-(17) fully determine the non-linear diamagnetic response of a clean SIN system to an externally applied magnetic field H .

It is easy to check that our result reduces to the already known ones in the corresponding limits. For instance, in the case of a perfectly transmitting SN interface ($R = 1 -$

$D = 0$) Eqs. (14)-(17) reproduce the non-linear response derived in Ref. 6. Another important limit is that of a small external field. In this case one can linearize the function G (17) in ϕ and find

$$j = -\frac{ek_F^2 T \phi}{\pi} \sum_{\omega_n > 0} \int_0^{\pi/2} \frac{d\theta \sin^3 \theta t^2(\theta) \cosh \chi_1}{\cos \theta [t^2(\theta) + \sinh^2 \chi_1]^{3/2}}, \quad (18)$$

This result was obtained in Ref. 14 with the aid of a different approach. It is interesting that at low temperatures $T \ll tv_F/d$ the transmission and reflection coefficients drop out and for any nonzero t the result (18) reduces to a simple formula

$$j = -\frac{ek_F^2 v_F \phi}{6\pi^2 d}, \quad (19)$$

which was initially derived⁶ for $t = 1$. At higher temperatures t does not anymore drop out of the final result. For instance, in the limit $T \gg v_F/d$ we obtain

$$j = -\frac{ek_F^2 v_F t_0^2 \phi}{2\pi^3 d} \left(\frac{v_F}{Td} \right) e^{-4\pi Td/v_F}, \quad (20)$$

where t_0 is the value of $t(\theta)$ at $\theta = 0$. In the case of low transmissions $t_0 \ll 1$ there exists an additional temperature interval $t_0 v_F/d \ll T \ll v_F/d$, where the current shows a power-law dependence on temperature:

$$j = -\frac{7\zeta(3)ek_F^2 v_F \phi}{64\pi^4 d} \left(\frac{v_F}{Td} \right)^2 \int_0^{\pi/2} d\theta \cos^2 \theta \sin^3 \theta t^2(\theta). \quad (21)$$

It is important to emphasize that the screening current in the N-metal is *uniform* in space and, hence, is a nonlocal function of the vector potential. Combining this result with the Maxwell equations one easily arrives at the self-consistency condition for the “phase” ϕ

$$\frac{\phi c}{e} = Hd^2 + \frac{8\pi}{3c} j(\phi) d^3. \quad (22)$$

In the linear response regime the current density j can be expressed in the form $j = -c^2 \phi / 8\pi e \lambda_N^2(T) d$. As soon as the effective length $\lambda_N(T)$ becomes small, $\lambda_N(T) \ll d$, the non-locality of the screening current turns out to be important. For instance, at $T \rightarrow 0$ we have $j = -3Hc/8\pi d$, the magnetic field penetrating into the normal layer becomes over-screened⁶, $B(x) = 3Hx/(2d) - H/2$, and the average magnetization of the N-layer reads $4\pi \mathcal{M} = -3H/4$.

III. MAGNETIC BREAKDOWN

The above Meissner state of our hybrid structure is thermodynamically favorable only in relatively small magnetic fields. At higher values of H the diamagnetic screening current gets suppressed and the magnetic field can freely penetrate into the N-layer. This magnetic

breakdown effect was experimentally studied by Mota and coworkers¹² and was also addressed theoretically^{7,13} in the case of perfectly transparent SN interfaces. In this section we will consider the magnetic breakdown of the Meissner effect in clean proximity structures with an arbitrary transmission of the SN interface. We will closely follow the strategy adopted in Ref. 13.

The free energy F of the current state (normalized per unit surface) can be recovered by means of the standard procedure which amounts to integrating the current (14) over an effective “coupling constant” λ

$$F(\phi) = -(\phi/2e) \int_0^1 j(\lambda\phi) d\lambda. \quad (23)$$

In the presence of an external magnetic field we have to minimize the Gibbs free energy

$$\mathcal{G}(T, H) = F(\phi) + \frac{1}{8\pi} \int_0^d dx (\partial_x A(x) - H)^2. \quad (24)$$

In the limit of low external fields $H < H_{\min}$ the diamagnetic solution described in the previous section is the only possible one. On the contrary, at high magnetic fields $H > H_{\max}$ this solution cannot be realized. In this case another solution of the Maxwell equations – which describes full penetration of the magnetic field into the N-layer – takes over. In the intermediate regime $H_{\min} < H < H_{\max}$ the diamagnetic and non-magnetic solutions may coexist. The first order phase transition between these two states occurs in this intermediate regime at a certain value of the magnetic field $H = H_b$ implying the magnetic breakdown of Meissner screening. Since this breakdown field H_b can be sufficiently large, the full nonlinear expression for the diamagnetic response, Eqs. (14)-(17), should be taken into account.

First let us briefly consider the case of a perfectly transparent SN interface $t \equiv 1$ and re-derive the results¹³. The “supercooling” and “superheating” fields H_{\min} and H_{\max} can easily be estimated from Eq. (22). Combining this equation with the expression for the screening current $j = -c^2\phi/8\pi e\lambda_N^2(T)d$ one readily finds $\phi \simeq 3\pi H\lambda_N^2(T)/\Phi_0$. Since the diamagnetic solution is possible only for small $\phi \ll 2\pi$, one obtains

$$H_{\max} \sim \Phi_0/\lambda_N^2(T). \quad (25)$$

For a non-magnetic solution the screening current is suppressed $j \approx 0$, in which case from Eq. (22) we find $\phi \simeq \pi H d^2/\Phi_0$. The vanishing screening current implies strong dephasing of Andreev states in the N-layer. This dephasing effect occurs for $\phi \gg 2\pi$. Hence, we get

$$H_{\min} \sim \Phi_0/d^2. \quad (26)$$

Now let us evaluate the breakdown field H_b . The Gibbs energy of a non-magnetic state \mathcal{G}_{nm} is minimized by the equation $\partial_x A = H$. Making use of Eqs. (14)-(17)

and (23,24) together with the inequality $\phi \gg 2\pi$ one finds

$$\mathcal{G}_{nm} = \frac{k_F^2 T}{\pi} \sum_{\omega_n > 0} \int_0^1 \ln \left[1 + \exp \left(-\frac{4\omega_n d}{\mu v_F} \right) \right] \mu d\mu, \quad (27)$$

where we denoted $\mu \equiv \cos \theta$.

The free energy of the diamagnetic state \mathcal{G}_{dm} can also be easily established. Assuming $\lambda_N(T) \ll d$ we obtain

$$\mathcal{G}_{dm} = 3H^2 d/32\pi. \quad (28)$$

The breakdown field H_b is derived from the condition $\mathcal{G}_{dm} = \mathcal{G}_{nm}$. In accordance with Ref. 13 one gets

$$H_b \simeq \frac{\sqrt{2}\Phi_0}{\pi\lambda_N(0)d} e^{-2\pi T d/v_F} \quad (29)$$

in the high temperature limit $T \gg v_F/d$ and

$$H_b \simeq \frac{\Phi_0}{6\lambda_N(0)d} \quad (30)$$

in the opposite limit of low temperatures $T \ll v_F/d$. We observe that for $\lambda_N(T) \ll d$ the breakdown field H_b is parametrically larger (smaller) than H_{\min} (H_{\max}), i.e. the phase transition indeed occurs in the intermediate regime $H_{\min} < H < H_{\max}$. The condition $\lambda_N(T^*) \sim d$ defines the temperature $T^* = (v_F/2\pi d) \ln(d/\lambda_N(0))$ below which the above picture remains valid. At higher temperatures $T \gtrsim T^*$ a continuous and reversible cross-over between the two states is expected¹³.

Let us now turn to the case of a non-ideal SN interface $t < 1$. The estimate (26) for the field H_{\min} remains the same while (25) should be modified. Below we will estimate H_{\max} and obtain the expression for H_b in three different temperature intervals.

High temperatures. In the limit $T \gg v_F/d$ Eqs. (14)-(17) yield

$$j = -\frac{ek_F^2 v_F t_0^2 \phi}{2\pi^3 d} \left(\frac{v_F}{Td} \right) e^{-4\pi T d/v_F} \exp \left[-\frac{v_F \phi^2}{2\pi T d} \right]. \quad (31)$$

Making use of this expression we evaluate the Gibbs energy and find H_b which is again defined by Eq. (29) multiplied by the factor t_0 . Analogously, the estimate (25) should now be multiplied by t_0^2 and the whole picture remains consistent at $t_0 \gtrsim \lambda_N(T)/d$.

Intermediate temperatures. In the limit $t_0 \ll 1$ it is possible to realize the condition $t_0 v_F/d \ll T \ll v_F/d$. In this intermediate temperature range we obtain

$$j = -i \frac{ek_F^2 v_F^2}{16\pi^4 T d^2} \int_0^{\pi/2} d\theta \int_0^{\pi/2} d\varphi \cos^2 \theta \sin^2 \theta \cos \varphi \times \\ t^2(\theta) \cos \chi_2 \left[\psi' \left(\frac{1+i\rho}{2} \right) - \psi' \left(\frac{1-i\rho}{2} \right) \right], \quad (32)$$

where $\psi(z) = \Gamma'(z)/\Gamma(z)$ is the digamma function and

$$\rho = \frac{v_F \cos \theta \sin \chi_2}{2\pi T d}. \quad (33)$$

In the limit $\phi \ll Td/v_F$ we recover the result (21), while for $\phi \gg Td/v_F$ the screening current *decreases* with increasing ϕ . For $Td/v_F \ll \phi \lesssim 1$ we get

$$j = -\frac{ek_F^2 v_F}{4\pi^2 d \phi} \int_0^1 \mu^2 t^2(\mu) d\mu. \quad (34)$$

The estimate for H_{\max} is obtained by combining Eqs. (21) and (22). In this regime H_{\max} turns out to be smaller than (25) by the factor $\sim t_0^2 v_F / Td$. The free energy of a non-magnetic state is also established easily. Making use of the condition $\phi \gg 2\pi$ and integrating the current (32) over λ in (23) we obtain

$$F = \frac{k_F^2 v_F}{8\pi^2 d} \int_0^1 d\mu \mu^2 t^2(\mu) \ln \kappa, \quad (35)$$

where $\kappa = \gamma_0 \mu v_F / (2\pi Td)$ and $\ln \gamma_0 \simeq 0.577$ is the Euler constant. The magnetic breakdown field H_b is obtained by comparing Eqs. (28) and (35). One finds

$$H_b = \frac{\Phi_0}{\pi \lambda_N(0) d} \left[\int_0^1 d\mu \mu^2 t^2(\mu) \ln \kappa \right]^{1/2}. \quad (36)$$

Thus, in the limit $t_0 \ll 1$ this field is strongly reduced as compared to the case of perfectly transparent SN interfaces (30).

Low temperatures. In the case $T \ll t_0 v_F / d$ the diamagnetic current takes the form

$$j = -\frac{ek_F^2 v_F}{\pi^3 d} \int_0^{\pi/2} d\theta \int_0^{\pi/2} d\varphi \sin^2 \theta \cos \theta \cos \varphi I(\chi_2, t). \quad (37)$$

Here $I(\chi_2, t(\theta))$ is a π -periodic function of χ_2 . For $|\chi_2| < \pi/2$ this function is defined as

$$I = \chi_2 - \text{sign } \chi_2 \Theta(|\sin \chi_2| - t) \arctan \frac{\sqrt{\sin^2 \chi_2 - t^2}}{\cos \chi_2}, \quad (38)$$

where $\Theta(x)$ is the Heaviside step function. The function $I(\chi_2, t)$ is depicted in Fig. 1 for different values of t . We observe that this function is linear in $\chi_2 \propto \phi$ only at $\phi \lesssim t$. For $t \ll \phi \lesssim 1$ we again recover the dependence (34).

The estimates for H_{\max} and H_b follow from the above expressions for the current. At $T \rightarrow 0$ the field H_{\max} turns out to be by the factor $\sim t_0$ smaller than (25) and H_b is again defined by Eq. (36) with $\kappa \simeq 1/t_0$ in this case. The above analysis remains consistent down to very small transmissions $t_0 \gtrsim \lambda_N(0)/d$.

The dependence of H_b on the transparency parameter t_0 in the limit $T \rightarrow 0$ was also calculated numerically from Eqs. (37,38) and the result is presented in Fig. 2. With a sufficient accuracy this dependence can be approximated by a simple formula

$$H_b(t_0)/H_b(1) \simeq t_0. \quad (39)$$

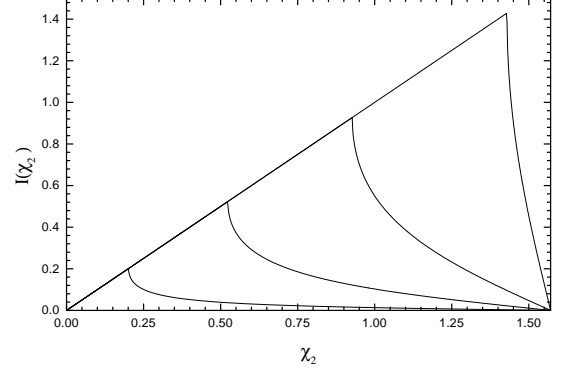


FIG. 1: The function $I(\chi_2)$ (38) for different values of $t = 0.2, 0.5, 0.8, 0.99$ (left to right). The initial slope remains the same for all curves.

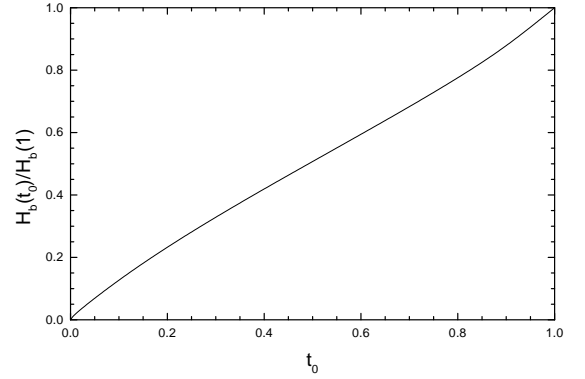


FIG. 2: The magnetic breakdown field H_b as a function of t_0 evaluated at $T = 0$ for the angle-dependent transmission of the form (40).

The above results for $H_b(t_0)$ can be used to extract information about the effective transmission of the SN interface from the measurements of the breakdown field H_b . A detailed comparison between theoretical predictions (derived at $t = 1$) and experimental data for H_b was carried out in Refs. 13,18. It was found in both papers that for comparatively clean samples the temperature dependence of the breakdown field is quite well described by a theory derived in the clean limit. At the same time the amplitude of H_b turned out to be somewhat smaller (typically by a factor $\sim 0.3 \div 0.6$) than that predicted in the ballistic case. One can attribute this discrepancy to the effect of electron scattering on non-magnetic impurities in the N-layer¹⁹. It follows from the above analysis that another possible reason for this suppression of H_b is a non-perfect transmission of the SN interface.

For instance, one can argue that the difference between theory (30) and experiment by a factor ≈ 0.56 found in Ref. 13 at low T could be due to the effect of the inter-

face transmission. Then from Fig. 2 one would recover $t_0 \approx 0.55$. In the most relevant case of a thin effective potential barrier at the SN interface the angle-dependent transmission is defined as

$$t(\theta) = \frac{t_0 \cos^2 \theta}{1 - t_0 \sin^2 \theta}, \quad (40)$$

in which case the above estimate for t_0 will translate into the value of the minimum interface reflectivity $R_{\min} \simeq 0.29$. Hence, one can conclude that SN interfaces in the experiments¹² were indeed highly transmissive as it was already assumed before by a number of authors. Of course, other factors, such as electron scattering on impurities in the N-layer, differences in geometry and corrections due to the term \mathbf{A}^2 neglected in Eq. (4), can cause additional minor discrepancies²⁰ between theoretical and experimental values of H_b and can slightly modify our estimate for R_{\min} . However, the above conclusion can hardly be affected by these minor modifications. In fact, the estimate for R_{\min} could only increase if electron scattering on impurities is taken into account in addition to the effect discussed here.

IV. APPLICABILITY OF QUASICLASSICS AND ABSENCE OF PARAMAGNETIC REENTRANCE

Finally let us briefly address the applicability of the quasiclassical Eilenberger approach for the problem in question. Solving the Eilenberger equations in the S- and N-metals and matching these solutions at the SN interface with the aid of the Zaitsev boundary conditions we have evaluated the screening current in the N-layer of our proximity system. The calculation is to much extent analogous to that presented in Appendix A of Ref. 15, therefore we will not go into details here. Although the result of this calculation does not exactly match with our general expression (13), it turns out to be *identical* to one presented by Eqs. (14)-(17). The latter result was obtained from (13) by means of averaging over all possible directions of the Fermi momentum or, equivalently, over the angle γ . Similarly to Ref. 15 the difference between the expressions for the current before and after this averaging is small in the parameter $1/k_F d$. We also note that in the case of highly transmitting interfaces, $R \rightarrow 0$, no averaging over γ is needed and Eq. (13) coincides exactly with the result of the quasiclassical analysis⁶.

The above calculation was carried out in the limit of large spacial dimensions L_y and L_z of our system respectively in y and z directions. For finite $L_{y,z}$ our results for the screening current acquire a small correction. Its magnitude can easily be estimated by rewriting the integral over the momentum directions in Eq. (13) as a sum over the conducting channels. Using further the Euler-Maclaurin summation formula

$$\frac{1}{2}F(a) + \sum_{n=1}^{\infty} F(a+n) \approx \int_a^{\infty} F(x)dx - \frac{1}{2}F'(a) \quad (41)$$

we observe that the correction to our results due to finite $L_{y,z}$ is small as $1/\sqrt{\mathcal{N}}$, where $\mathcal{N} \sim k_F^2 L_y L_z$ is the effective number of conducting channels in the x -direction.

The parameters $1/k_F d$ and $1/\sqrt{\mathcal{N}}$, therefore, control the accuracy of the quasiclassical Eilenberger approach in our problem. Both these parameters are very small since they contain the ratio between the Fermi wavelength and at least one of the system dimensions. For instance, for the systems studied in Refs. 3,4,5,12 we estimate²¹ $1/k_F d \lesssim 10^{-4}$ and $1/\sqrt{\mathcal{N}} \lesssim 10^{-6}$. Hence, one can conclude that the Eilenberger formalism remains very accurate in this case and no significant physics is missing within this formalism at any temperature and at any interface transmission.

The above estimates also indicate that within our model no paramagnetic reentrance effect can be expected in SN proximity cylinders. Indeed in the limit $L_y \gg d$ the difference between the slab and cylinder geometries is negligible and our results will be directly applicable to the latter geometry if we identify L_y with the circumference of the cylinder. Then the correction from all electron states within the energy interval $\sim \Delta$ from the Fermi energy (obviously this correction also includes the contribution of the low energy glancing states⁸) is small as $\sim 1/\sqrt{\mathcal{N}}$.

Although the conclusion about the absence of paramagnetic reentrance was obtained here only in the clean limit and for specularly reflecting interfaces, this conclusion should apply in the presence of impurities as well. In fact, a purely ballistic situation appears to be most favorable for the low energy glancing states. As soon as a finite electron mean free path l is introduced a proximity induced minigap $\sim v_F \min(l^{-1}, l/d^2)$ will develop in the normal metal²² and there will be no glancing states in the system at all.

We also note that our present consideration does not include the contribution of the electron states with energies smaller than $\epsilon_F - \Delta$. This contribution to the current (which has nothing to do with superconductivity and glancing states) can be roughly estimated with the aid of the standard results for the persistent currents in normal metallic cylinders, see, e.g., Ref. [23]. Making use of these results, in the limit of small magnetic fields one can assume $I_{PC} \sim e(v_F/L_y)k_F^{3/2}d^{1/2}L_z\phi$. The relative magnitude of this contribution is also much smaller than the proximity induced diamagnetic current studied here. At $T \rightarrow 0$ and for relatively high interface transmissions one finds $I_{PC}/jdL_z \sim [d/k_F L_y^2]^{1/2}$. For typical experimental parameters^{3,4,5} this ratio is of order $\sim 10^{-4}$, i.e. in this case no noticeable correction to our results could be expected either.

In summary, with the aid of the microscopic Gor'kov equations we have derived a general expression for the non-linear diamagnetic current in a clean superconductor-insulator-normal metal structure with an arbitrary interface transmission. We have demonstrated that at low temperatures the magnetic breakdown field H_b is suppressed linearly with the transmission of the in-

ulating barrier. This observation enables one to obtain information about the quality of inter-metallic interfaces from measurements of H_b . We have also compared our results with those derived within the quasiclassical Eilenberger approach and found the latter approach to be extremely accurate for the systems under consideration at all relevant temperatures down to $T = 0$ and at any in-

terface transmission.

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- ¹ C.J. Lambert and R. Raimondi, J. Phys. Cond. Mat. **10**, 901 (1998).
 - ² W. Belzig, F.K. Wilhelm, C. Bruder, G. Schön, and A.D. Zaikin, Superlattices and Microstructures **25**, 1251 (1999).
 - ³ P. Visani, A.C. Mota, and A. Pollini, Phys. Rev. Lett. **65**, 1514 (1990).
 - ⁴ F.B. Müller-Allinger and A.C. Mota, Phys. Rev. Lett. **84**, 3161 (2000).
 - ⁵ F.B. Müller-Allinger and A.C. Mota, Phys. Rev. B **62**, R6120 (2000).
 - ⁶ A.D. Zaikin, Solid State Commun. **41**, 533 (1982).
 - ⁷ W. Belzig, C. Bruder, and G. Schön, Phys. Rev. B **53**, 5727 (1996); W. Belzig, C. Bruder, and A.L. Fauchère, *ibid.* **58**, 14531 (1998).
 - ⁸ C. Bruder and Y. Imry, Phys. Rev. Lett. **80**, 5782 (1998).
 - ⁹ A.L. Fauchère, W. Belzig, and G. Blatter, Phys. Rev. Lett. **82**, 3336 (1999).
 - ¹⁰ K. Maki and S. Haas, cond-mat/0003413.
 - ¹¹ A.L. Fauchère, V. Geshkenbein, and G. Blatter, Phys. Rev. Lett. **82**, 1796 (1999); C. Bruder and Y. Imry, *ibid.*, **82**, 1797 (1999).
 - ¹² A.C. Mota, P. Visani, and A. Pollini, J. Low Temp. Phys. **76**, 465 (1989); A.C. Mota, P. Visani, A. Pollini, and K. Aupke, Physica B **197**, 95 (1994).
 - ¹³ A.L. Fauchère and G. Blatter, Phys. Rev. B **56**, 14102 (1997).
 - ¹⁴ S. Higashitani and K. Nagai, J. Phys. Soc. Jpn. **64**, 549 (1995).
 - ¹⁵ A.V. Galaktionov and A.D. Zaikin, Phys. Rev. B **65**, 184507 (2002).
 - ¹⁶ A.A. Abrikosov, L.P. Gorkov, and I.Ye. Dzyaloshinski. *Quantum Field Theoretical Methods in Statistical Physics* (Second Edition, Pergamon, Oxford, 1965).
 - ¹⁷ L.D. Landau and E.M. Lifshitz, *Quantum Mechanics* (Pergamon, Oxford, 1962).
 - ¹⁸ F.B. Müller-Allinger, A.C. Mota, and W. Belzig, Phys. Rev. B **59**, 8887 (1999).
 - ¹⁹ It was demonstrated in Ref. 18 that a quantitative theoretical description of the experimentally observed diamagnetic susceptibility is possible only provided the effect of a finite electron mean free path in the N-metal is taken into account.
 - ²⁰ Using Eq. (29) multiplied by $t_0 \approx 0.56$ one can obtain an estimate for H_b at high temperatures $T \gg v_F/d$. The corresponding result will coincide with one presented by the solid curve in Fig.3 of Ref. 13. However, the experimental data in this high temperature regime turn out to be smaller than this estimate by a constant factor ≈ 0.65 .
 - ²¹ In our estimates we use typical values $d \sim 5\mu\text{m}$, $L_y \sim 100\mu\text{m}$ and $L_z \sim 1\text{mm}$.
 - ²² S. Pilgram, W. Belzig, and C. Bruder, Phys. Rev. B **62**, 12462 (2000).
 - ²³ H.F. Cheung, Y. Gefen, and E.K. Riedel, IBM J. Res. Develop. **32**, 359 (1988).